**SETON HALL UNIVERSITY**

**TWENTYSECOND ANNUAL**

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**MATHEMATICS COMPETITION**

1. Find the sum of the cubes of the five smallest positive integers,

2. Tim has 36 coins, some nickels and the rest pennies. He has a total of 88¢. How many fewer nickels than pennies has he?

3. The angles of a (convex) quadrilateral are in the ratio 4:5:7:8. Find the degree measure of the largest angle in the quadrilateral.

4. It is now between 10 o'clock and 11 o'clock. In 12 minutes it will be 5 times as many minutes before 11 o'clock as it is now after 10 o'clock. In what number of minutes will it first be 11 o'clock?

5. A particle moves in an elliptical path on a coordinate plane. At time $t$ seconds (where $t$ is a nonnegative real number) the particle is located at the point $(x, y)$, with $x$ and $y$ given by the parametric equations $x = 5 - 2\cos(t),\ y = 3 + 4\sin(t)$. Find the time $t$ ($t$ measured in seconds), $0 \le t \le 2\pi$, for which the particle is located at the point (5, -1). Give the answer in exact form, do not approximate.

6. Find the value of $a^5(a+6) + 5a^3(3a+4) + 3a(5a+2)$ given that $a + 1 = \sqrt[6]{7}$.

7. Triangles $A$ and $B$ are both right triangles with one side (not the hypotenuse) of length 40 feet. If the perimeter of Triangle $A$ is 90 feet and the length of the hypotenuse of Triangle $B$ exceeds the length of the hypotenuse in Triangle $A$ by the length of the shortest side in Triangle $A$, find the sum of the lengths of the hypotenuse in Triangle $A$ and the shortest side in Triangle $B$.

8. How many different groups of 6 children can be selected from 10 boys and 6 girls if at least 3 boys and at most 3 girls are to be chosen?

9. In 3-dimensional space line $m$ intersects plane $P$ in point $T$, and $m$ is perpendicular to $P$. The points in space which are 2 inches or 4 inches or 6 inches from plane $P$ and are also 5 inches or 7 inches or 9 inches from point $T$ lie on circles. Find the sum of the squares of the radius lengths of the circles.

10. Six integers from the set $H = \{1, 2, 3, \ldots, 50\}$ are chosen at random, without replacement, and arranged in increasing order. Find the probability that the six numbers chosen (after rearrangement) form an arithmetic progression. Give the answer as a reduced fraction.

11. Find all nonnegative values of $x$ less than $2\pi$ for which $2\cos^2(x) - \sin(x) < 2$. Give your answer in interval form.

12. A group of 20 people bought tickets to a play; the cost per ticket was either \$35, \$20, \$12 or \$10. The total cost of the 20 tickets purchased was \$401. Half of the group members purchased either the \$20 or \$12 tickets. The amount spent for the \$35 and \$10 tickets exceeded the amount spent for the \$20 and \$12 tickets by \$49. How many more group members purchased \$20 tickets than purchased \$10 tickets?

13. Andy can complete a certain task alone in the same number of hours that Charlie can complete the task (alone) and in 2/3 of the number of hours that it takes Brad to complete the task (alone). Andy worked on the task alone for a few hours, then Brad continued (alone) for 3 times as long as Andy had worked, and the task was finished by Charlie (alone) in half the time that Brad worked on it. The entire task can be completed by all three working on it in only ¼ hour longer than it took Charlie to finish the task. In how many hours can Andy and Brad working together complete the entire task?

14. A train leaves Hill Station and runs at a speed of $r$ miles per hour (mph). A second train, going 18 mph faster than the first train, left Hill Station 2 hours later and followed the first train. After going a distance of 168 miles, the first train had mechanical difficulties and was forced to proceed at 2/3 its original rate. The second train maintained its original speed the entire time; it passed the first train 1½ hours after the first train reduced its rate. Find the distance (in miles) from Hill Station to the point at which the second train passed the first.

15. Denote the difference in the sum of the squares of the $N$ smallest even positive integers and the sum of the squares of the $N$ smallest odd positive odd integers by $S(N)$. Denote the sum of the cubes of the $M$ smallest positive numbers by $T(M)$. For a certain (small) value of $M$, and for $N = 3M$, the value of $S(N) = 3T(M)$. Find the value of $S(N+M)$.

16. In triangle ABC, altitude CD is 8 feet long and is perpendicular to side AB at D (with D between A and B). Line segment CE intersects side AB at E and CE bisects angle ACB and line segment DE is 1 foot long. If side AB is 448/15 feet long, find the perimeter of triangle ABC.